Exploring Critical Points And Critical Regions

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PhD in Mathematical Engineering
Doctoral Seminar 2

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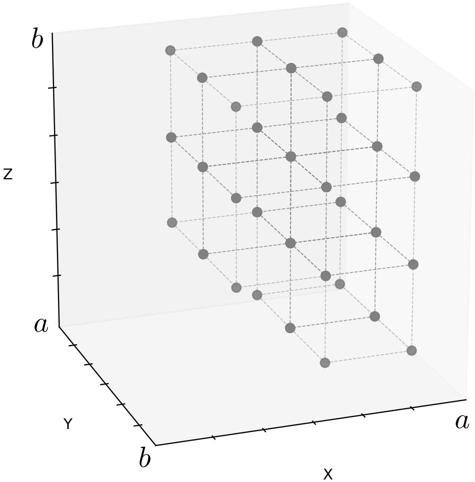
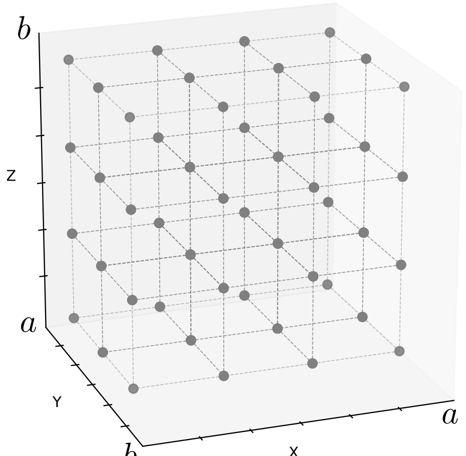
Grid: $x, y, z \in [a, b]$, n partitions



Regular and Critical Points - First step

Local classification

$$f(x, y, z)$$



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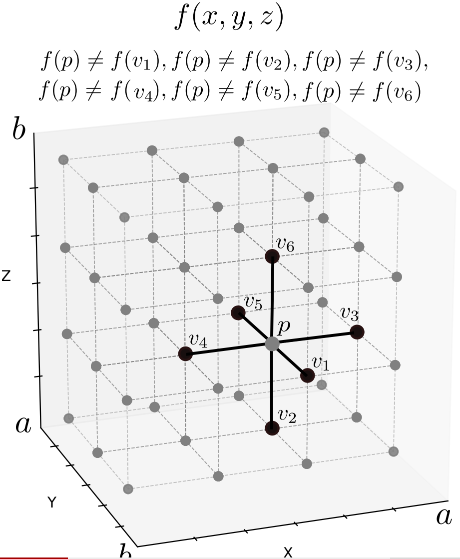

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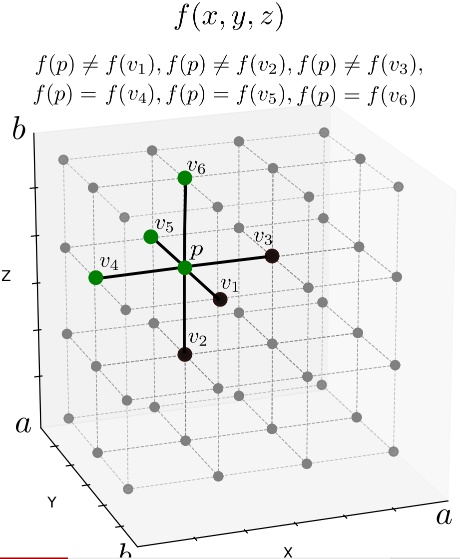

Regular and Critical Points - First step

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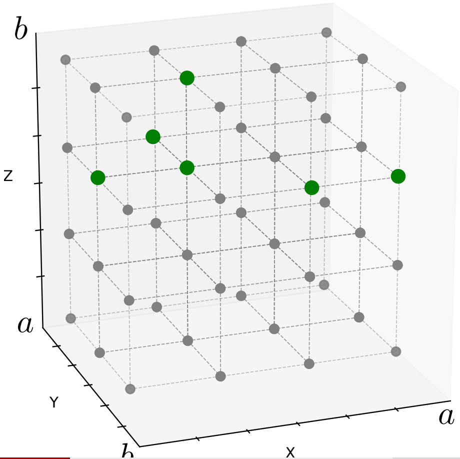
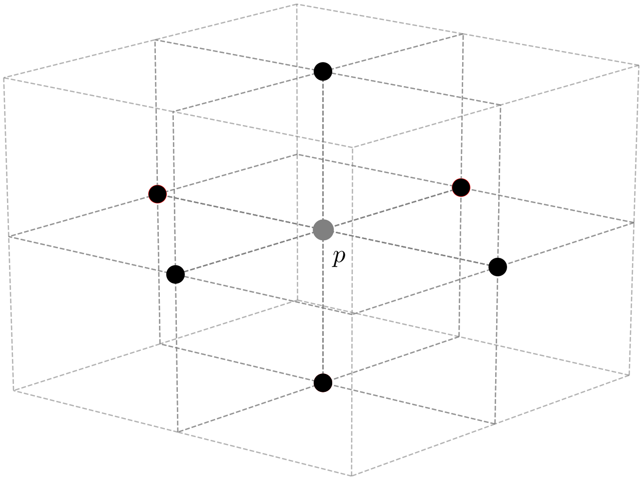
$$f(x, y, z)$$



Regular and Critical Points

Local classification



Regular and Critical Points

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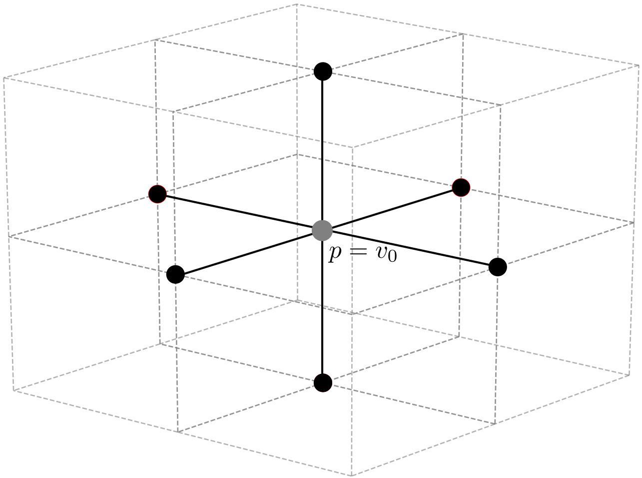

Regular and Critical Points

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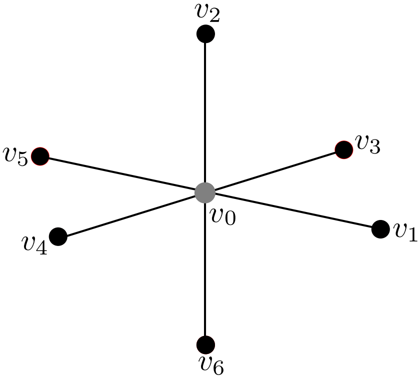

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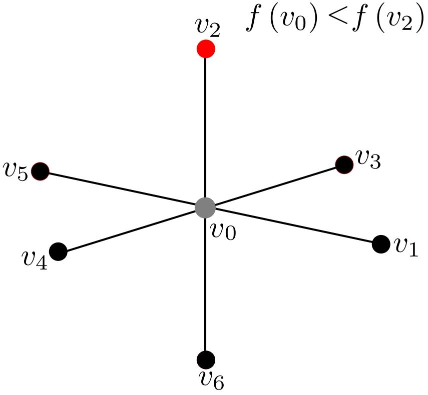

Regular and Critical Points

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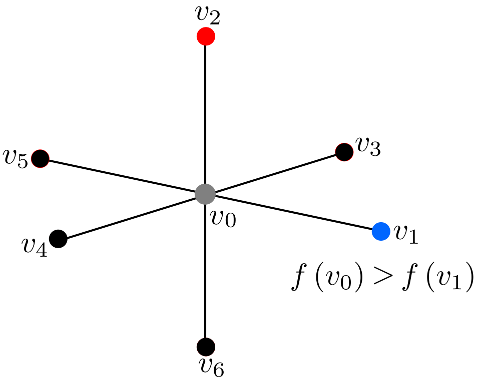

Regular and Critical Points

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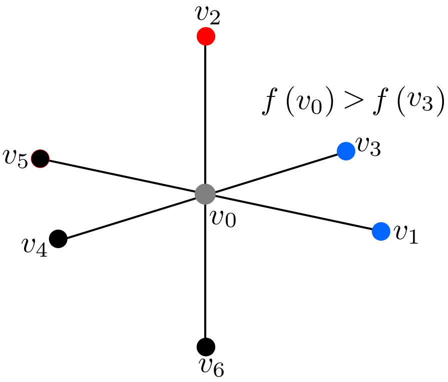

Regular and Critical Points

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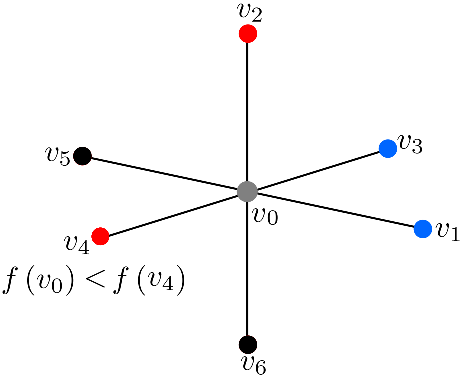

Regular and Critical Points

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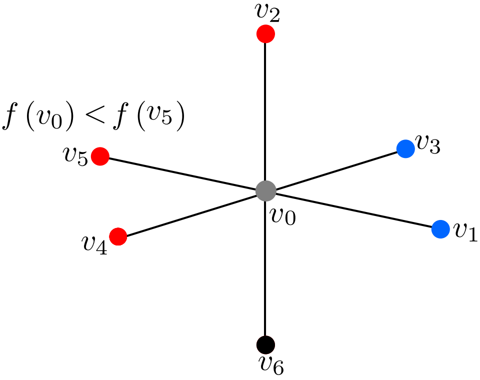

Regular and Critical Points

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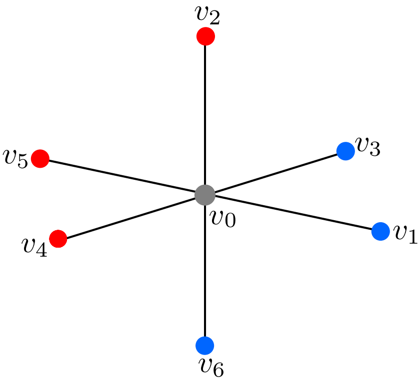
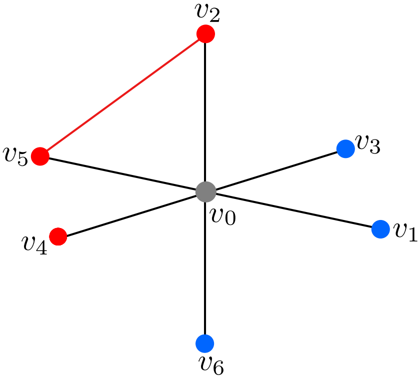
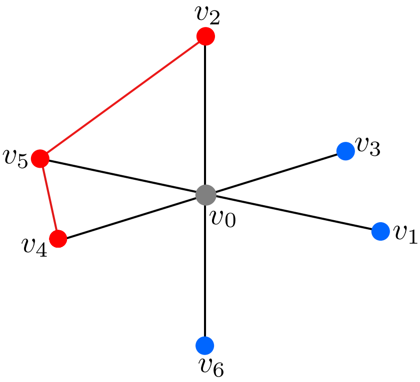


$$f(v_0) > f(v_6)$$

Local classification



Local classification



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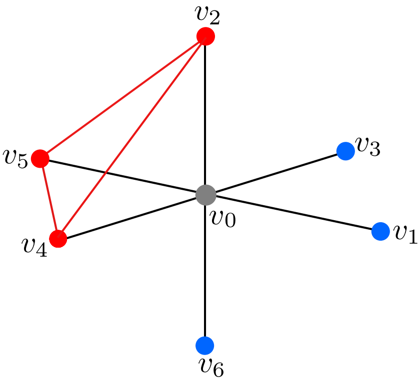

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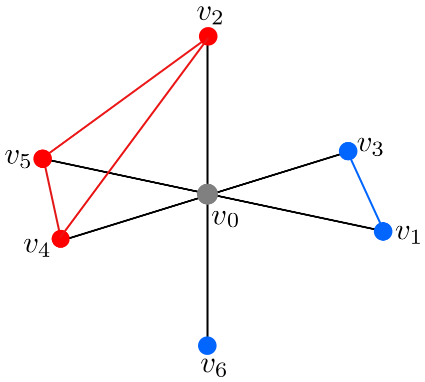

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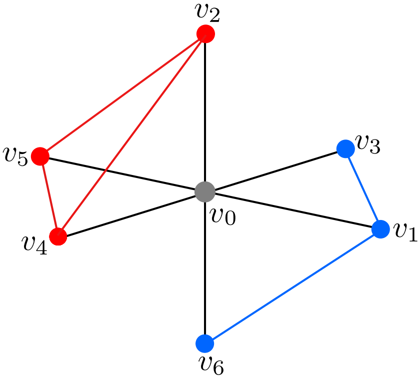

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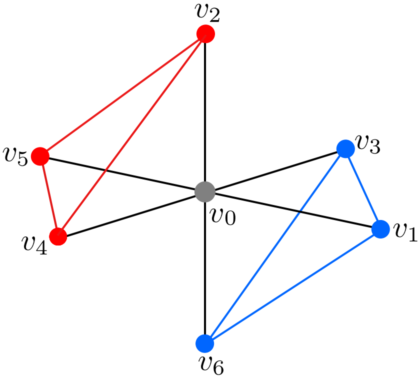

Regular and Critical Points

Definition 1

Let $M \subset \mathbb{R}^3$ be a mesh and $F : M \to \mathbb{R}$ be a C^0- continuous function that is $C^\infty-$ continuous function in each grid cell. A point $x \in \mathbb{R}^3$ is called regular or ordinary, minimum, maximum, saddle, extended minimum, extended maximum, extended saddle, or flat point of F, if for all $\varepsilon > 0$ there exists a neighborhood $U \subset U_\varepsilon(x)$ with the following properties:

(Weber, Scheuermann, Hagen, and Hamann 2002)

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Regular and Critical Points

Definition 1

If

$$\bigcup_{i=1}^{n_p} P_i$$ is a partition of the preimage of $[F(x), \infty)$ in $U - x$

into "positive" conected components,

$$\bigcup_{j=1}^{n_n} N_i$$ is a partition of the preimage of $(-\infty, F(x)]$ in $U - x$

into "negative" connected components and

$$\bigcup_{k=1}^{n_z} Z_k$$ is the partition of the preimage of $\{F(x)\}$ in $U - \{x\}$

into "zero set" connected components, then

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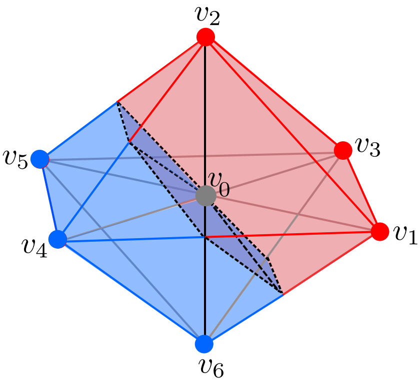
Regular and Critical Points

Regular point

$$n_p = n_n = n_z = 1$$



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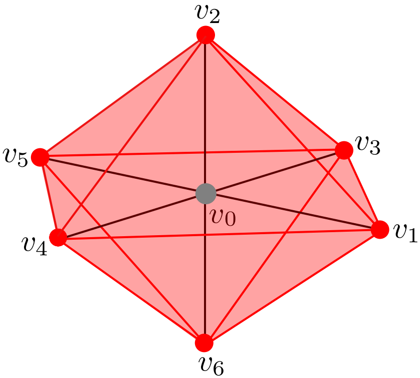
Regular and Critical Points

Minimum

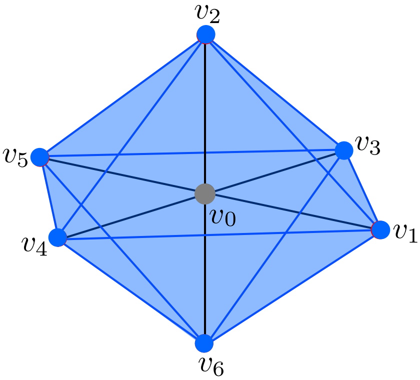
$n_p = 1$ and $n_n, n_z = 0$



Regular and Critical Points

Maximum

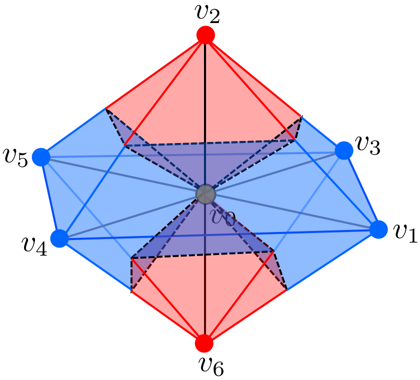
$n_n = 1$ and $n_p, n_z = 0$



Regular and Critical Points

Saddle

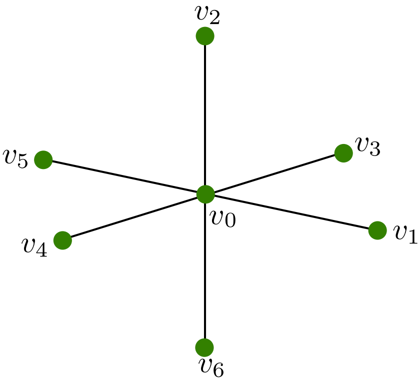
$$n_p + n_n > 2, n_n, n_p \geq 1, n_z > 1$$



Regular and Critical Points

Flat point

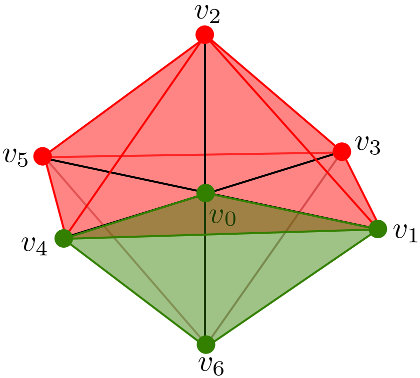
$n_z = 1$ and $n_p = n_n = 0$



Regular and Critical Points

Extended minimum

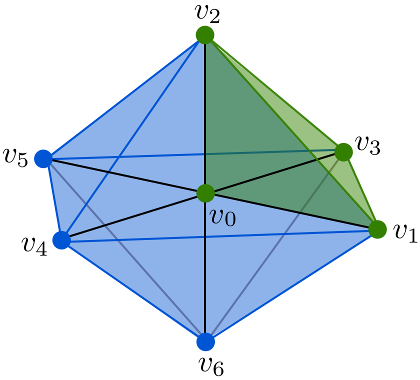
$$n_p = 1, n_n = 0, n_z \geq 1$$



Regular and Critical Points

Extended maximum

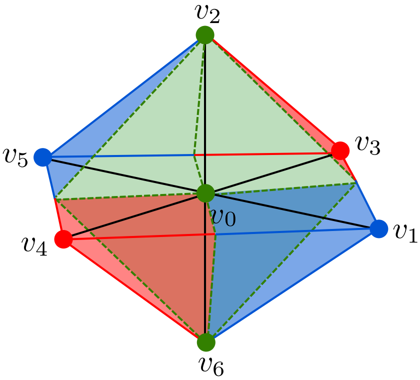
$$n_n = 1, n_p = 0, n_z \geq 1$$



Regular and Critical Points

Extended saddle

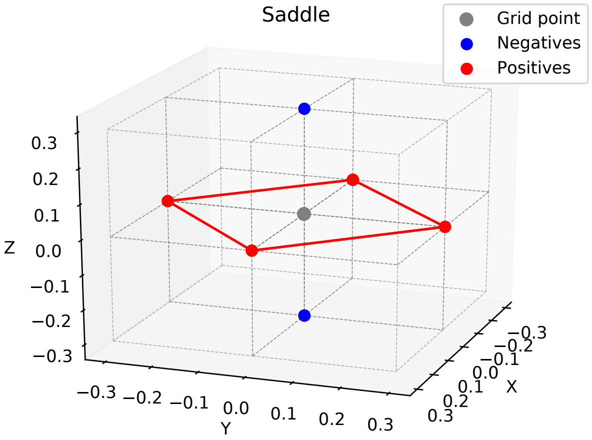
$$n_p + n_n > 2, n_z = 1$$



Regular and Critical Points

Example

$$f(x,y,z) = x^2 + y^2 + 0.5z^3 - 0.4975z^2 - 0.0025$$
$$[-1.5, 1.5],\ n = 10$$



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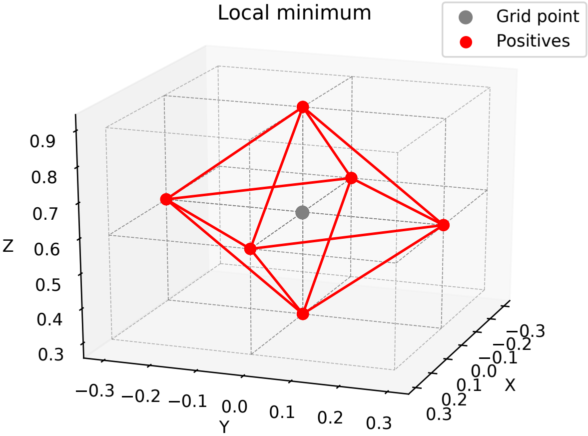
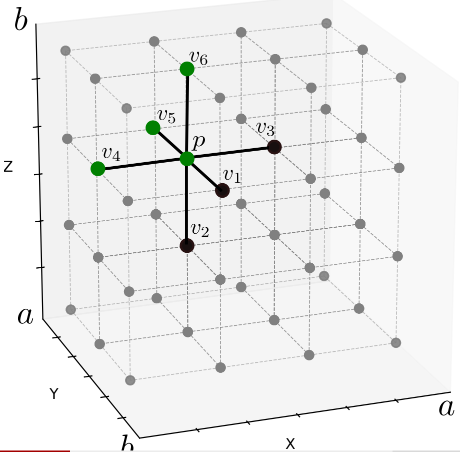


Regular and Critical Regions - Second step

Classification Regions

$$f(x, y, z)$$

$$f(p) \neq f(v_1), f(p) \neq f(v_2), f(p) \neq f(v_3),$$
$$f(p) = f(v_4), f(p) = f(v_5), f(p) = f(v_6)$$



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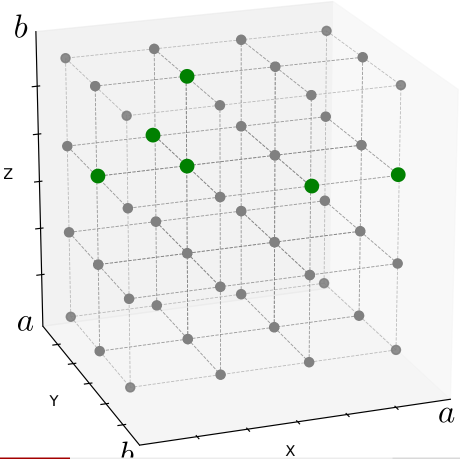
Regular and Critical Regions - Second step

Classification Regions

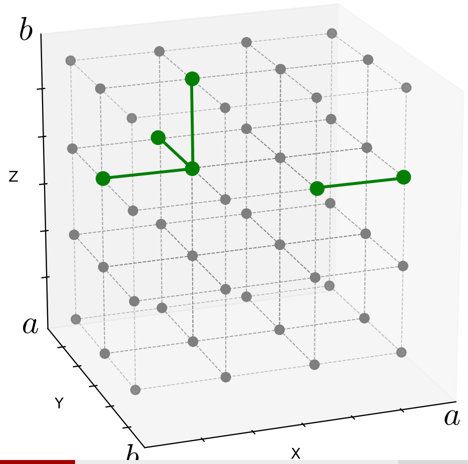
$$f(x, y, z)$$



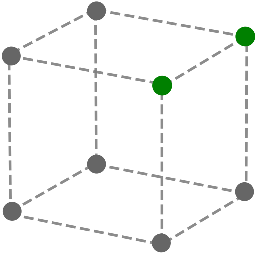
Regular and Critical Regions - Second step

Classification Regions

$$f(x,y,z)$$

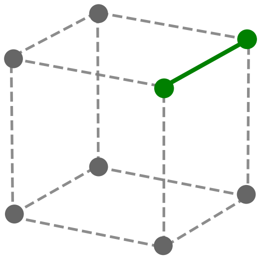


Regular and Critical Regions

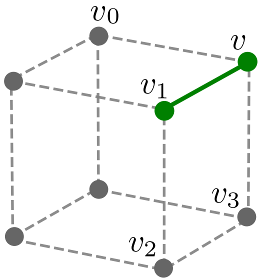


Regular and Critical Regions

Classification Region

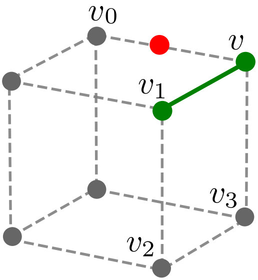


Regular and Critical Regions



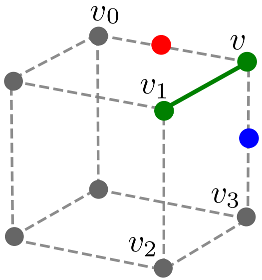
Regular and Critical Regions

$$f\left(v_0\right) > f\left(v\right)$$



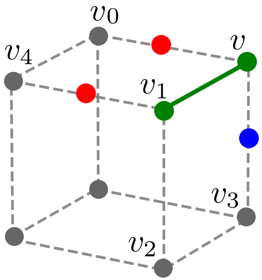
Regular and Critical Regions

$f(v_3) < f(v)$



Regular and Critical Regions

$f(v_4) > f(v_1)$



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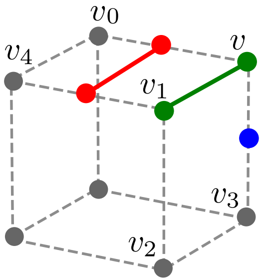

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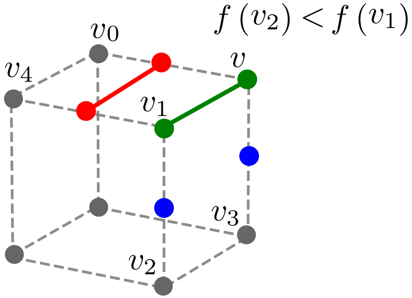

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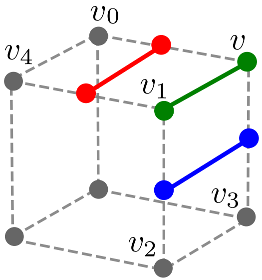

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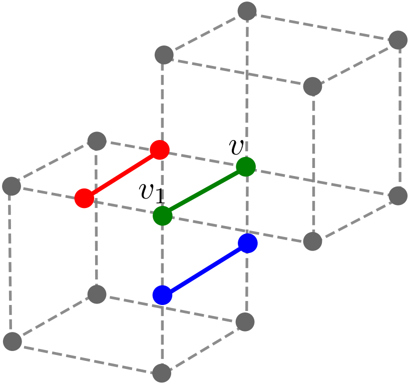

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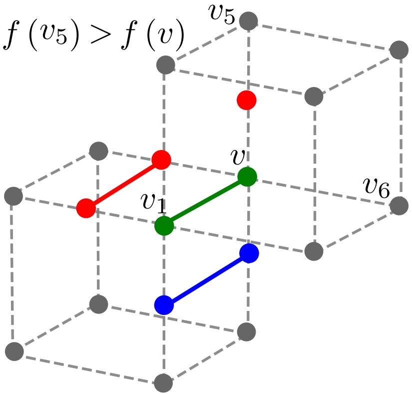

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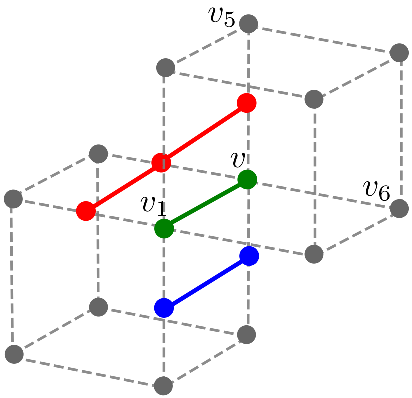

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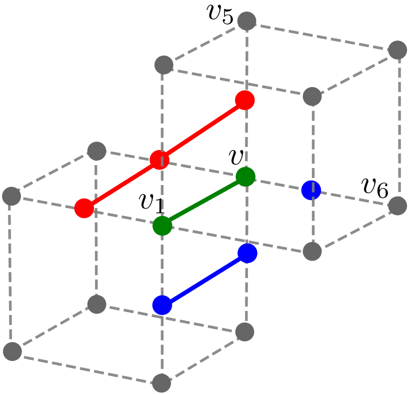

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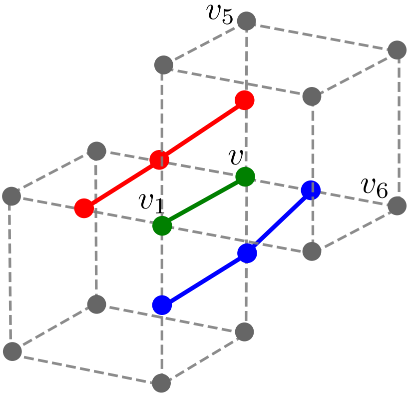

Regular and Critical Regions

Definition 2

Let $M \subset \mathbb{R}^3$ be a mesh and $F : M \to \mathbb{R}$ be a C^0- continuous function that is $C^\infty-$ continuous function in each grid cell.
A classification region $R \subset \mathbb{R}^3$ is called regular , minimum, maximum, saddle of F, if for all $\varepsilon > 0$ there exists a neighborhood $U \subset U_\varepsilon(R)$ with the following properties:

(Weber, Scheuermann, and Hamann 2003)

Regular and Critical Regions

Definition 2

If

$$\bigcup_{i=1}^{n_p} P_i$$ is a partition of the preimage of $[F(R), \infty)$ in $U - R$

into "positive" conected components,

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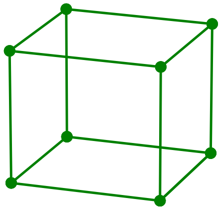
$$\bigcup_{k=1}^{n_z} Z_k$$ is the partition of the preimage of $\{F(R)\}$ in $U - R$

into "zero set" connected components, then

Regular and Critical Regions

Minimum region

$n_p \geq 1$ and $n_n = n_z = 0$



Regular and Critical Regions

Minimum region

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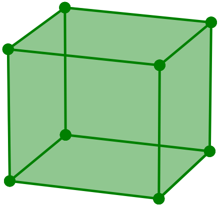

Regular and Critical Regions

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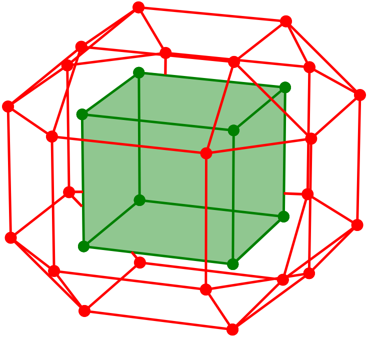

Regular and Critical Regions

Maximum region

$n_n \geq 1$ and $n_p = n_z = 0$

Regular and Critical Regions

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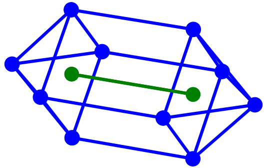
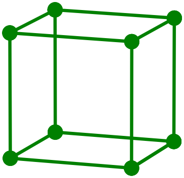


Regular and Critical Regions

Saddle region

$n_p + n_n > 2$ and $n_z = 1$



Saddle region

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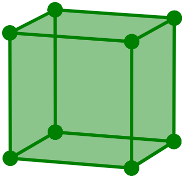
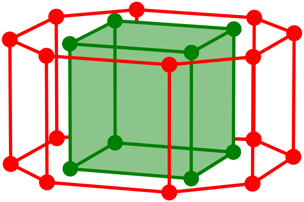


Regular and Critical Regions

Saddle region

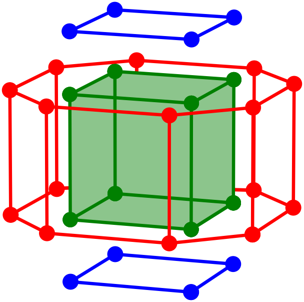
$n_p + n_n > 2$ and $n_z = 1$



Regular and Critical Regions

Saddle region

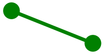
$n_p + n_n > 2$ and $n_z = 1$



Regular and Critical Regions

Regular region

$$n_p = n_n = n_z = 1$$



Regular and Critical Regions

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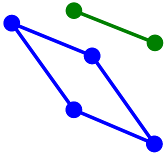

Regular region

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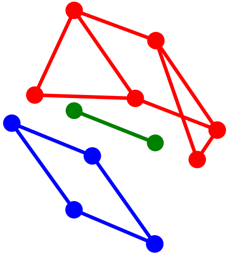
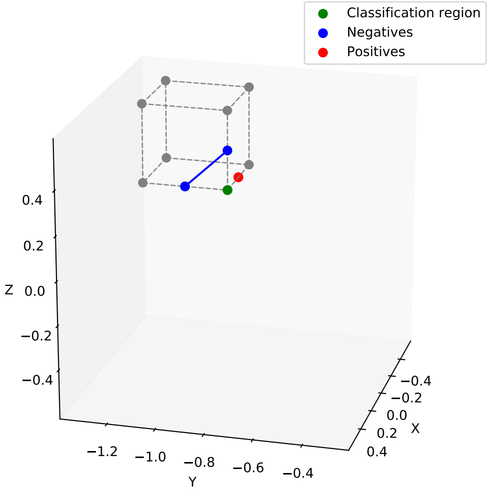


Example

$$f(x, y, z) = \left(3x^2 + 2y^2 + z^2\right) e^{-x^2 - y^2 - z^2}$$

Grid: $[-2, 2]$, points: 12^3. Classification region.



Example

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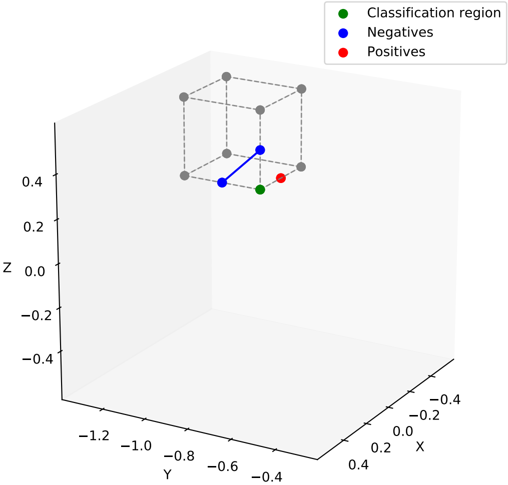

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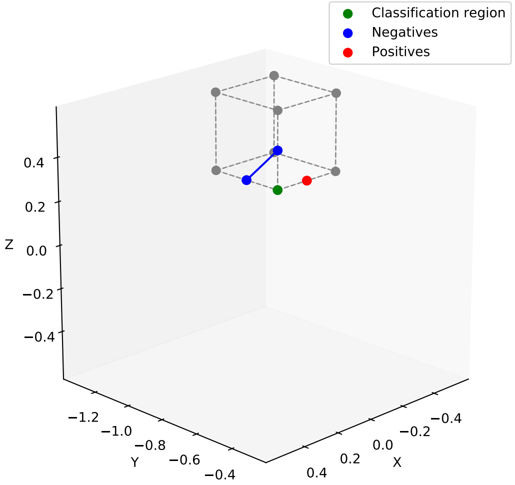

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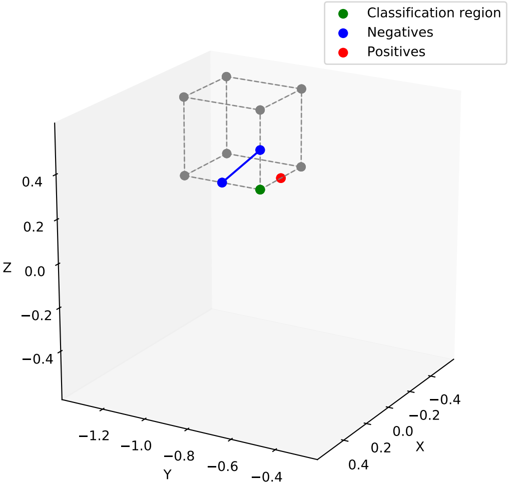

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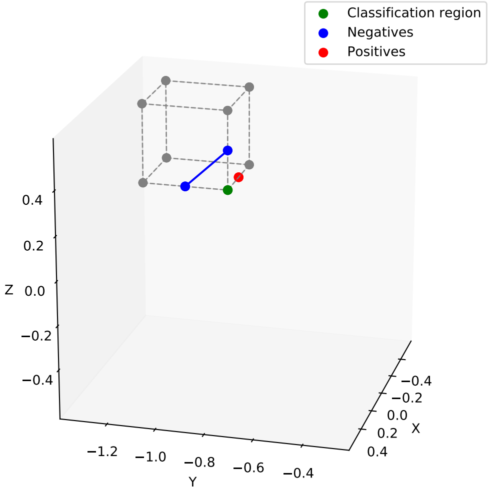

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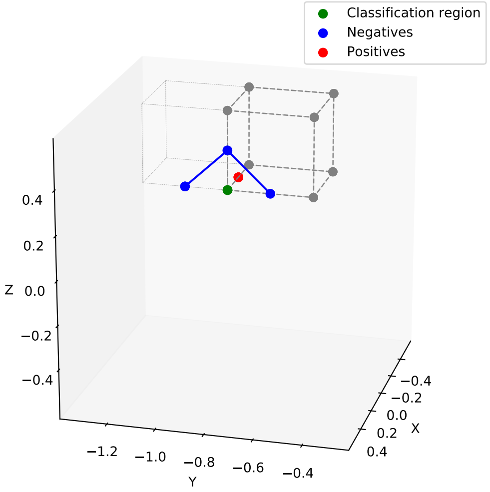

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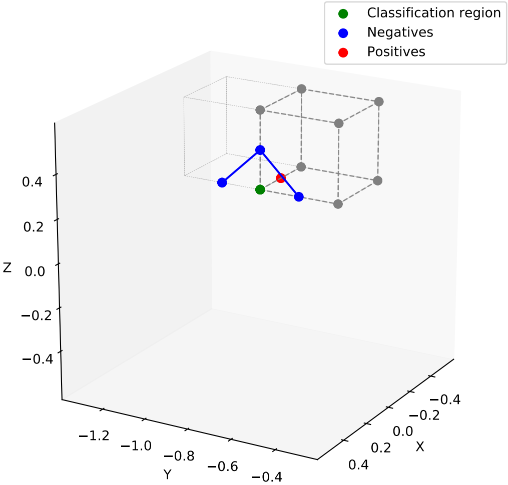

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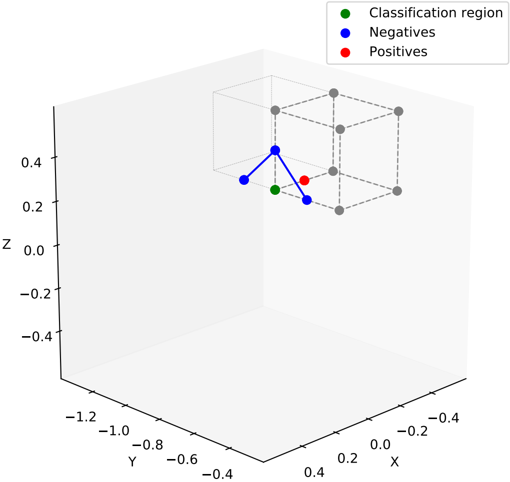

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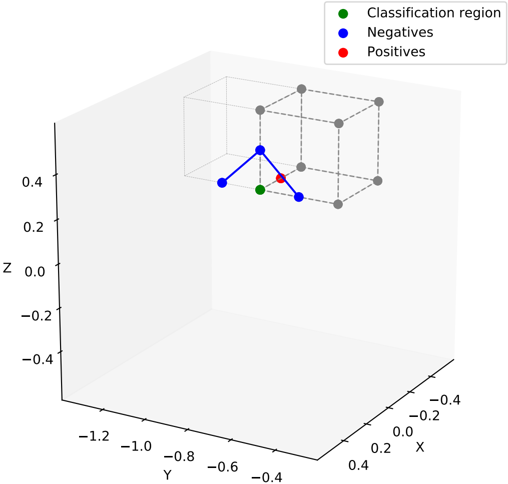

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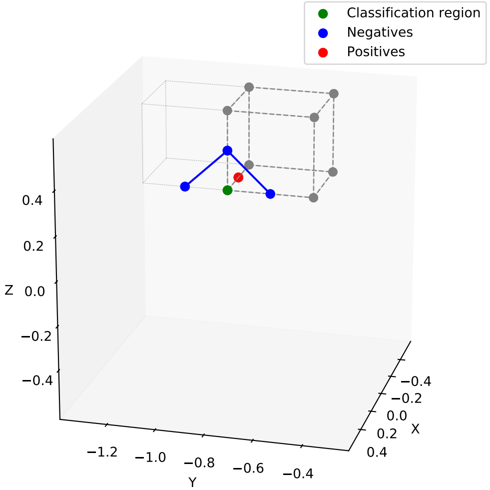

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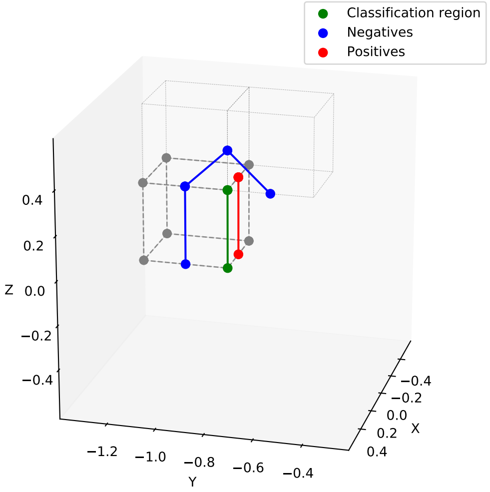

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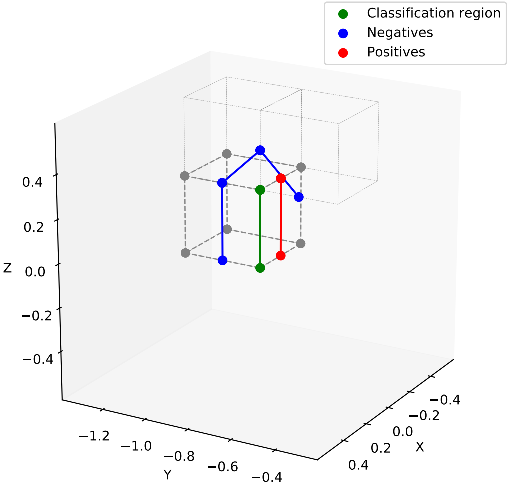

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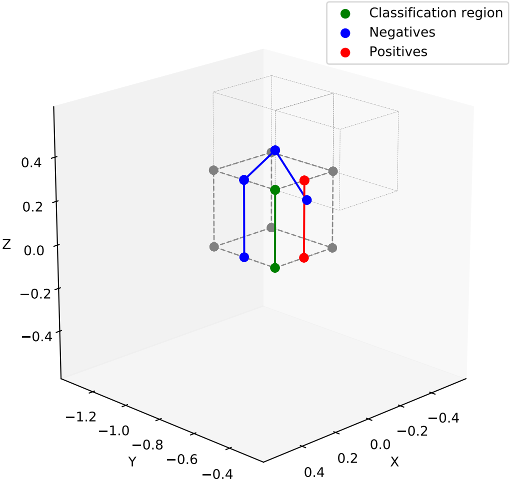

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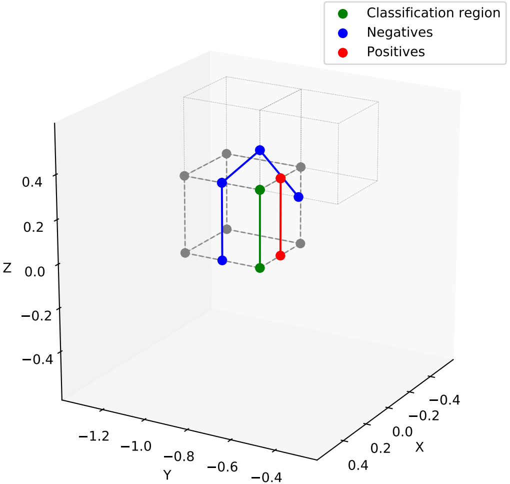

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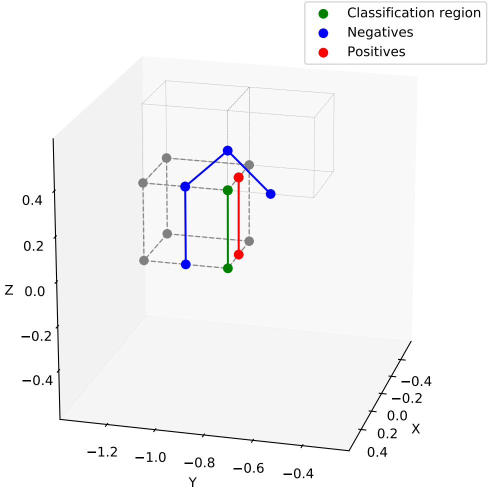

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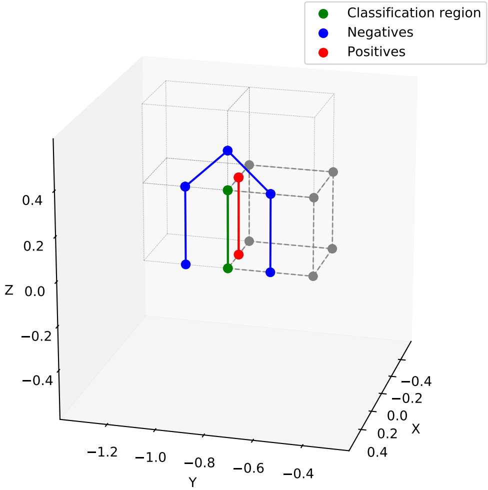

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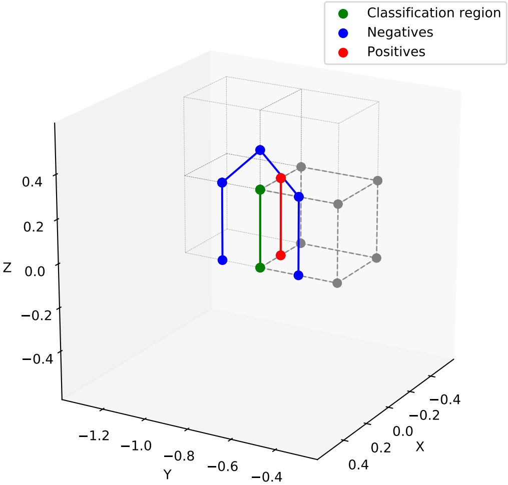

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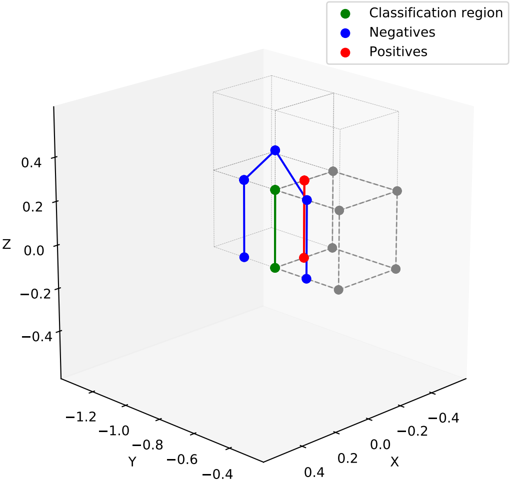

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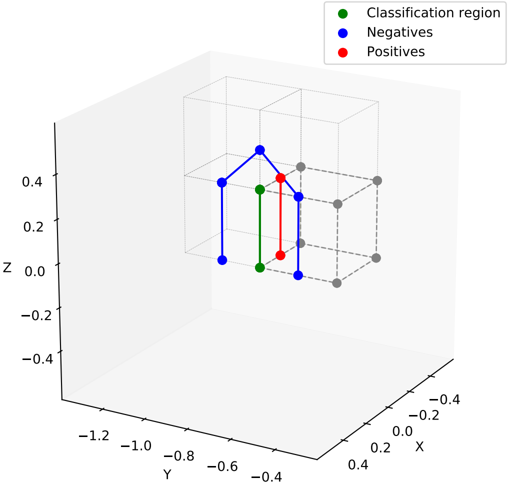

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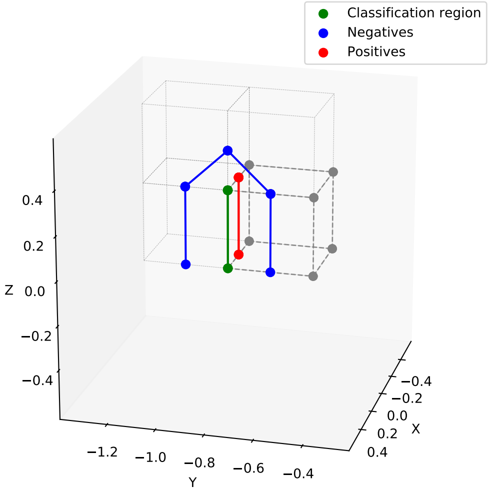

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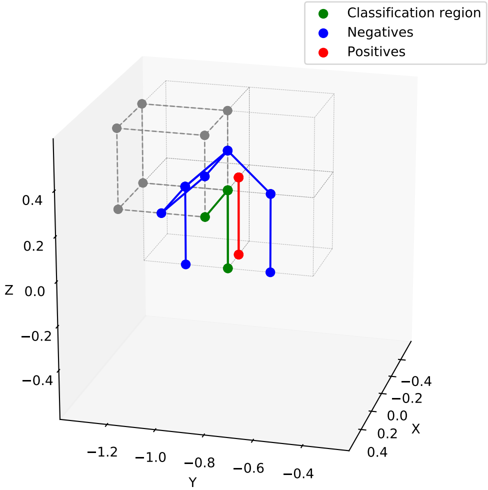

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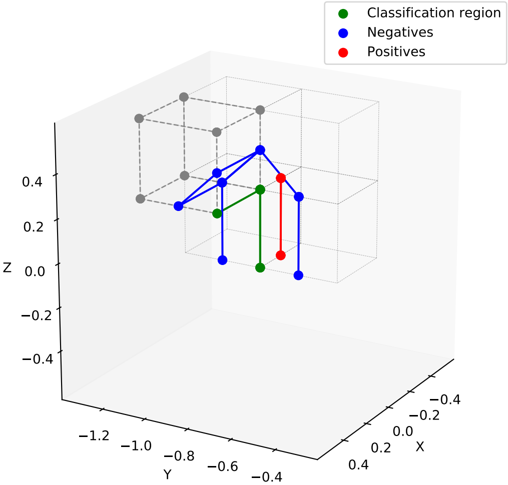

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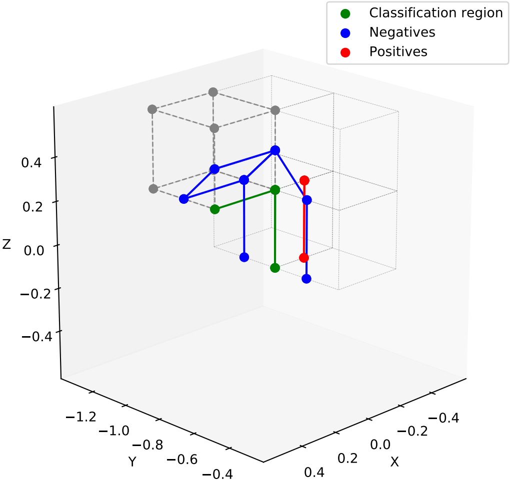

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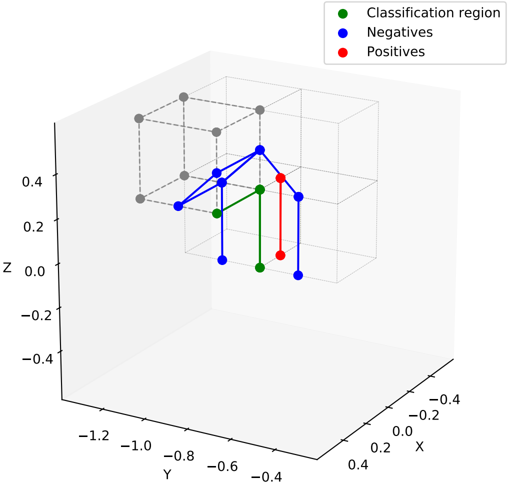

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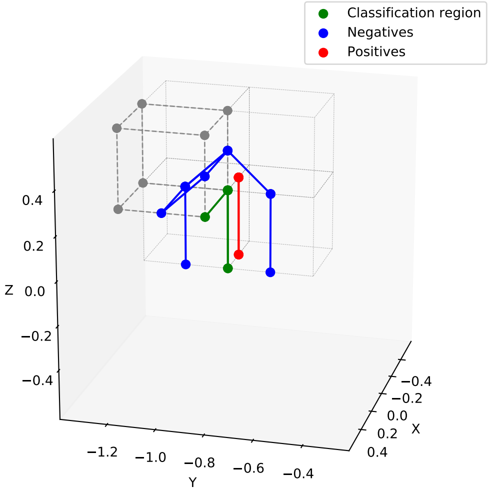

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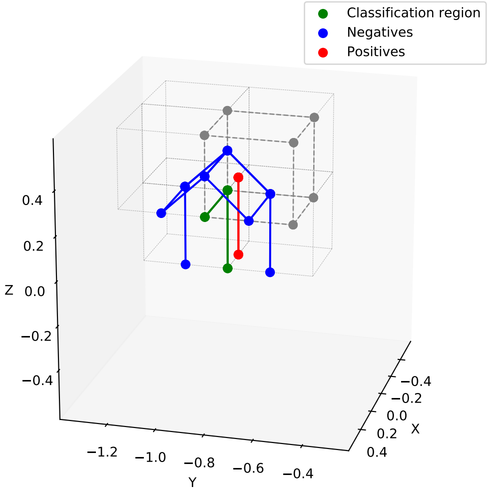

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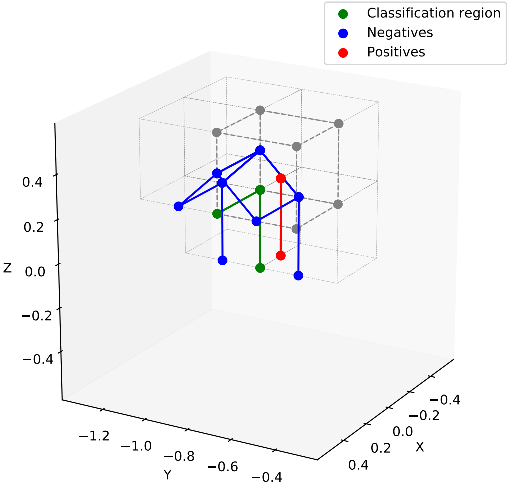

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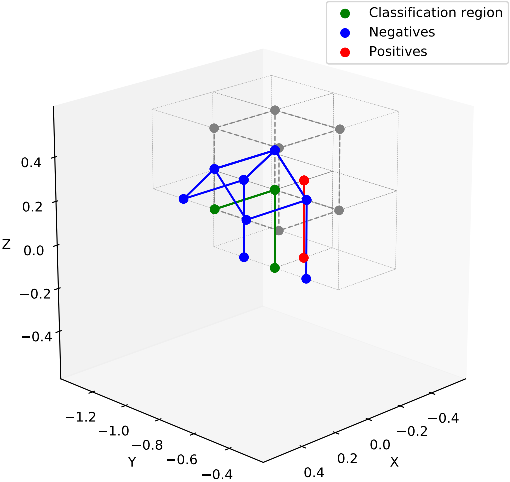

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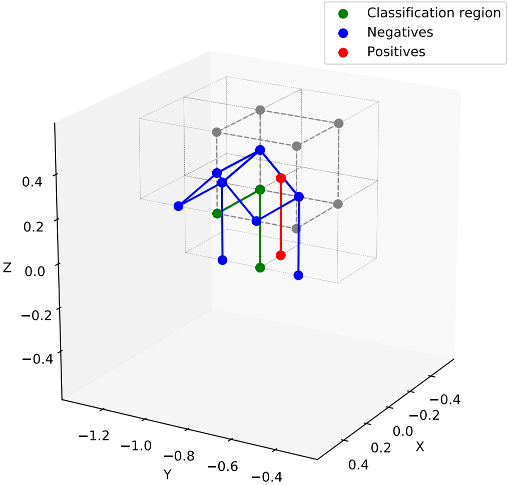

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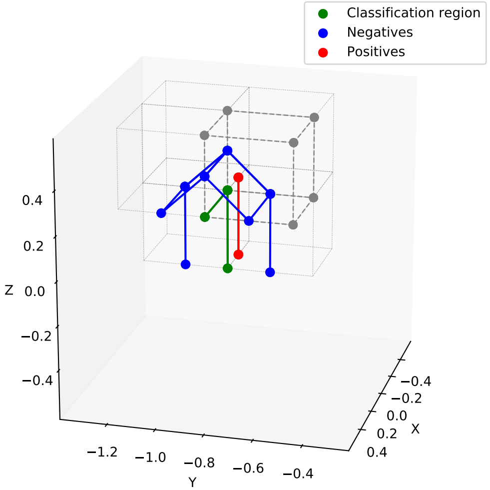

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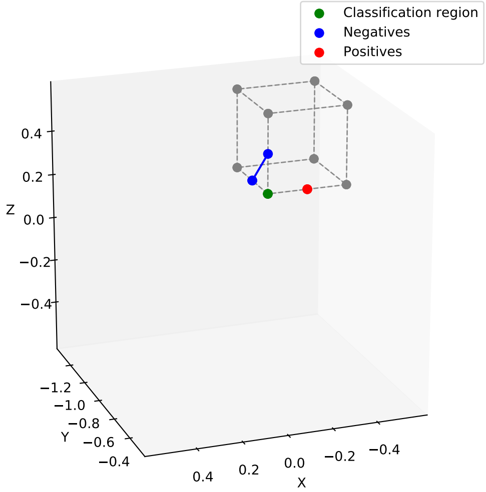

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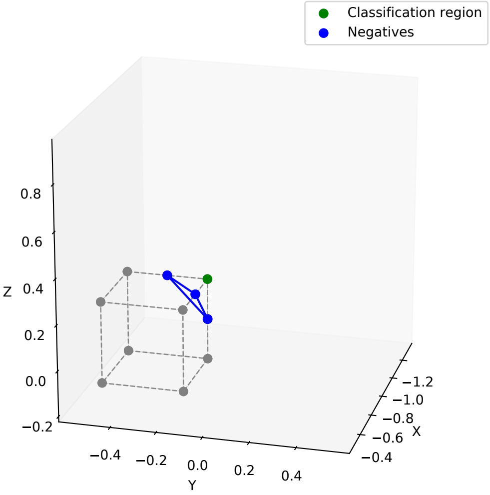

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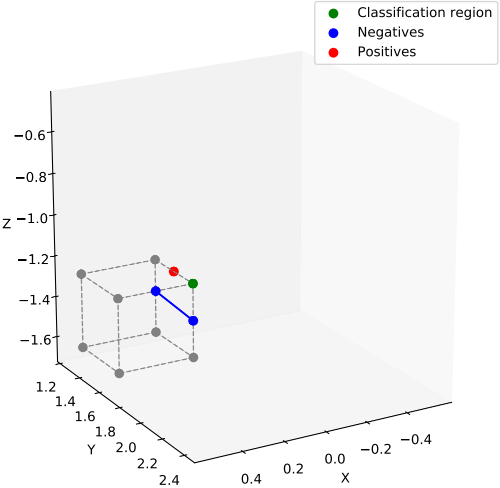

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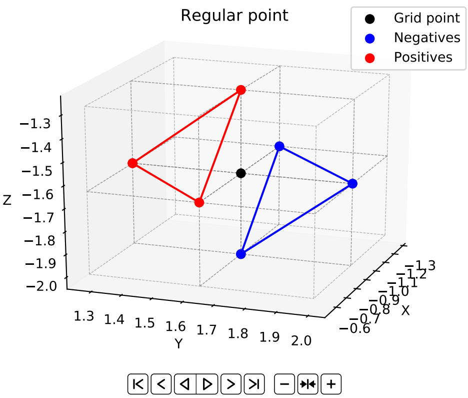

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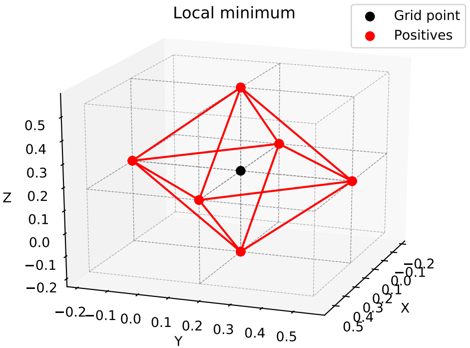
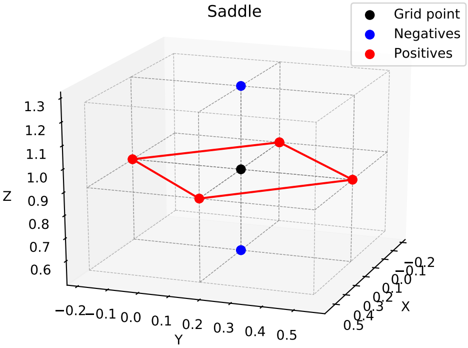
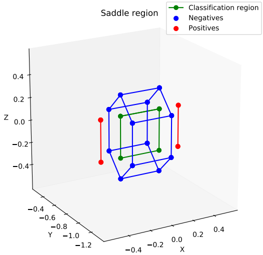


Classification local point



4 critical regions and 1 critical point.



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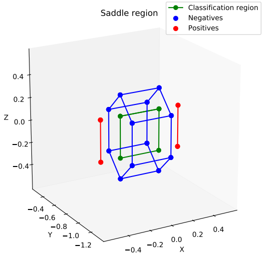

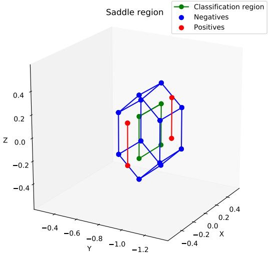

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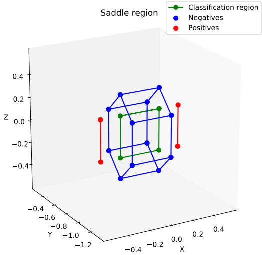

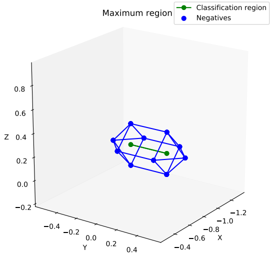

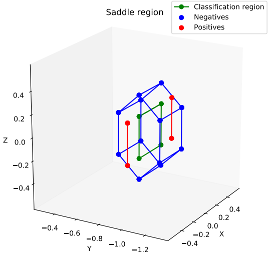

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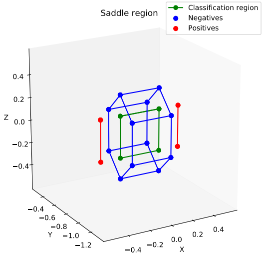

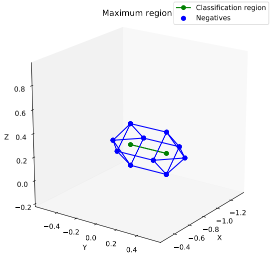

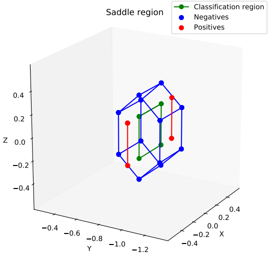

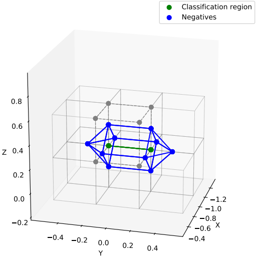

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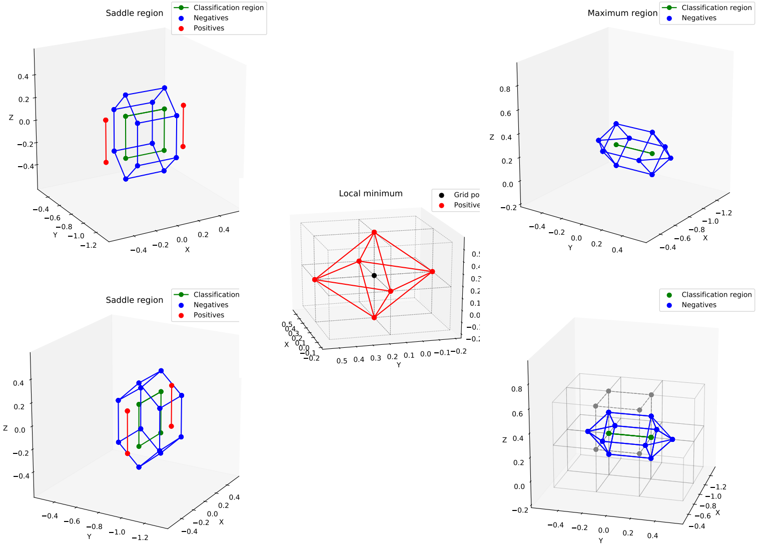

Bibliography

Weber, G. H., G. Scheuermann, H. Hagen, and B. Hamann (2002).
Exploring scalar fields using critical isovalues.
In *Proceedings of the conference on Visualization'02*, pp. 171–178. IEEE Computer Society.

Weber, G. H., G. Scheuermann, and B. Hamann (2003).
Detecting critical regions in scalar fields.
In *VisSym*, Volume 3, pp. 85–94.

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